## MATHEMATICS D

## Paper 4024/11 <br> Paper 11

## Key messages

To do well in this paper, candidates need to demonstrate that they have a good understanding across the whole syllabus. As it is a non-calculator paper, candidates need to be competent in basic numeracy skills, particularly in multiplication and division of whole numbers as well as decimals and be able to convert from one form to another. Candidates need to learn necessary formulae and facts. All working should be shown, and answers clearly written in the appropriate answer space.

## General comments

The majority of candidates had sufficient time to complete the paper. The paper gave all candidates an opportunity to demonstrate their strengths and weaknesses.

Many candidates showed clear working. Written figures such as 4 and 9, and also 1 and 7, were sometimes very similar and therefore difficult to distinguish when reviewing responses.

It is important for candidates to read the questions on the paper carefully, especially when instructions are given to write numbers to 1 significant figure for example, or to give answers in their simplest form. Some candidates dd not follow these instructions. It is also important for candidates to check they have copied numbers correctly from the question and also when transferring their answer from their working to the answer line.

Candidates generally showed good skills in basic numeracy. Arithmetic slips were seen quite frequently, so candidates should take care to avoid these by checking their working carefully. Candidates showed good skills in managing fractions. Some candidates found questions that required manipulating standard form more challenging. Candidates dealt with basic algebra well in the questions on factorising, simplifying and solving the linear equation. The questions on finding angles in parallel lines and circle geometry were handled well, as was constructing the angle bisector but candidates struggled to give the correct bearings in Question 13. Some candidates needed to improve their skills in multiplying matrices. Few candidates demonstrated a strong understanding when finding the median from a frequency table that contained similar numerical values.

The questions that candidates found most difficult were recognising the nets of a cube (Question 12), regions defined by inequalities (Question 16), transformations (Question 20), equation of a perpendicular line (Question 21), prime factors (Question 24) and similar triangles (Question 25).

## Comments on specific questions

## Question 1

(a) A very high proportion of candidates were able to subtract the fractions correctly.
(b) A large majority of candidates gave the correct answer. A few arithmetic errors were made when subtracting 0.36 and some candidates did not follow the correct order of operations. The most common error was losing the place value when multiplying $1.2 \times 0.3$ resulting in 3.6. This was often followed by $3.6-2=1.6$ or $2-3.6=-1.6$.

## Question 2

(a) Most candidates recognised the factorisation involved the difference of two squares and factorised correctly. Those who did not recognise the method often tried to take a common factor of 4 or 1 from the two terms.
(b) This part was very well answered by most candidates. Those who did not score full marks often managed to score one mark for a correct partial factorisation. Errors made at this stage usually involved minus signs when factorising or when re-ordering the terms.

## Question 3

(a) Almost all candidates were able to solve the linear equation. Some errors were made with signs when collecting terms over the equals sign.
(b) This question was well answered with most candidates scoring full marks. A common incorrect answer was $7-y$ which resulted from the incorrect expansion of the bracket $-3(3-y)=-9-3 y$. Following the expansion of the bracket, nearly all candidates collected their terms correctly.

## Question 4

(a) Many candidates were able to round the decimal 3456.789 correctly to the nearest 100 . Common errors included putting in zeros after the decimal point, rounding to the nearest 10, with some candidates also losing the place value giving the answer as 346 .
(b) Most candidates evaluated the square root correctly but 40000 was also a very common incorrect answer and some gave the answer 4000.
(c) The majority of candidates were able to write $3^{-1}$ as $\frac{1}{3}$ and proceeded to subtract this from 1 correctly, with many taking the longer route of finding a common denominator. Some thought $3^{-1}=-2$ and a common incorrect answer came from $1-0.3=0.7$.

## Question 5

This question was answered well by most candidates. Some started with $y$ proportional to $x$ which did not score any marks. Others made an arithmetic error in the latter stages, evaluating $3^{3}$ as 9 .

## Question 6

(a) Candidates understood what this question required but arithmetic errors were frequent. Those who worked entirely in months sometimes made errors when trying to convert their answer back into years and months. Others subtracted 1 year 8 months from 17 years 5 months as if they were working in base 10, giving 15 years 7 months as a common incorrect answer.
(b) Most candidates answered this correctly. Some converted 2 hours correctly to 120 minutes but forgot to add on the extra 18 minutes. A few arithmetic errors were made converting 2 hours 18 mins to 138 mins. Other errors were also seen in the process of cancelling $\frac{48}{138}$ to its simplest form, in which a few candidates reached $\frac{24}{69}$ but did not carry out the final cancellation.

## Question 7

(a) Most candidates completed the frequency column correctly. A few gave one incorrect value. Candidates should check their frequencies add up to the total given in the question. Many did not know how to use the middle column of the table as an aid and filled in the total frequencies there. If they did no further work to contradict this, the mark was awarded.
(b) Candidates were aware that the probabilities should add to 1 and calculated the correct answer.

## Question 8

Many correct responses were given for this question on rounding to one significant figure to give an estimated value. Some candidates rounded 6013 to 6 . Others did not correctly multiply $6000 \times 0.04$, often getting the place value incorrect, and 24000 was the most common error. Most candidates used the correct value of 3 for the square root. A few candidates did not estimate any of the numbers and tried to calculate an exact answer.

## Question 9

(a) Many candidates found this question on manipulating a calculation into standard form challenging. Incorrect answers were often seen from candidates who had shown no working out. Some of those who did show working, often lost place value although some did reach $4.3 \times 100$ and many got as far as writing $100^{2}=10000$. Many final answers included powers of 10 but were not in standard form.
(b) Many candidates also found this question on manipulating a calculation into standard form challenging. Some managed to reach $0.6 \times 10^{10}$ correctly to score a method mark. Many candidates evaluated $1.2 \div 2$ as 0.6 giving answers of the form $0.6 \times 10^{n}$ for various incorrect values of $n$ while others multiplied to give $2.4 \times 10^{n}$, also scoring no marks. Mistakes were most often made with the evaluation of $10^{7} \div 10^{-3}$ with $10^{4}$ seen regularly.

## Question 10

(a) (i) There were many correct answers but misunderstandings of how to find the mode were common. Errors included choosing 6, the frequency value for the mode. The number 4 was commonly seen in the table but also the highest number of goals scored, so some candidates used this and 0 goals to give 4 as the value for the range.
(ii) Candidates were less successful finding the median than the mode. Some knew to look for the value in the $(n+1)^{\text {th }}$ position while others wrote a list of all the goals scored in order to pick the middle value. As in the previous part, the most common incorrect answer was 4. This was the middle number in the row containing the frequencies, which happened to be in descending order of size.
(b) While many candidates were able to give the correct solution for this problem-solving question involving the mean masses of 10 football players, errors were equally frequent. A common error was to simply add the two masses 75 and 60 and to divide by 2 or 10 . Some candidates divided the correct total mass by 2 . Other candidates made arithmetic errors when multiplying $6 \times 80$.

## Question 11

(a) Many candidates gave the correct answer, often from using the longer method of finding all the angles inside the triangle rather than just using co-interior angles. Some candidates made arithmetic errors when subtracting $123^{\circ}$ from $180^{\circ}$. A few candidates believed that the bottom angle in the triangle was $123^{\circ}$ and used angles on a line $180^{\circ}-123^{\circ}-40^{\circ}$ to get an incorrect answer. A few gave the answer as $40^{\circ}$ applying opposite angles to the $40^{\circ}$ given.
(b) This part was also answered very well. Those who had (a) correct nearly always scored here too. Many others recovered as a result of correctly finding the angles inside the triangle using the angles given on the diagram rather than using their previous incorrect answer with the method of angles on a line.

## Question 12

Only stronger candidates were able to identify the 3 correct nets for a cube. Most other candidates were able to recognise $A$ as a correct answer but often incorrectly included $D$, which only had 5 faces. Various other incorrect combinations were given.

## Question 13

(a) Many candidates produced correct solutions to this question showing clear calculations. However, many did not know the correct approach. Some incorrectly tried to incorporate the ratio aspect of the question into their solution, with calculations such as $\frac{3}{10} \times 360$ or $\frac{3}{10} \times 140$. Others tried to use angles on a line by drawing extra horizontal lines on the diagram, making no correct progress.
(b) While a minority of candidates found the correct bearing, many different incorrect answers were seen. Candidates need to be aware that bearings must only be given as three figure numbers measured clockwise from North and that notation such as N 70 W will not score marks.
(c) Again, only a minority gave the correct bearing and many different incorrect answers were seen. These included $70^{\circ}$ (the bearing of A from O ) and $110^{\circ}$ (the co-interior angle from $70^{\circ}$ ), both of which were seen regularly.

## Question 14

(a) Some candidates were able to shade the correct regions in the Venn diagram. However, a range of different incorrect answers were seen.
(b) (i) This question proved challenging for many candidates. Various incorrect values were seen based on subtracting and adding combinations of the given values 42,30 and 20 . Some candidates simply subtracted the 30 and 20 and gave 10 as their answer, the difference between the numbers of people who speak Spanish and French.
(ii) Only the strongest candidates answered this part correctly. Common incorrect answers were 8, 10 and 0 . In both parts of this question a few candidates drew a Venn diagram but not all were filled in correctly.

## Question 15

(a) The majority of candidates were able to construct the angle bisector accurately showing all construction arcs.
(b) Candidates did less well in answering this part. Many candidates did not understand that a line parallel to $A C$ was required. Some constructed a bisector of one or more of the sides of the triangle. Many diagrams displayed arcs and/or circles in various places, some centred at $A$ and $C$ but many at more random locations.

## Question 16

(a) Candidates found this question difficult with only stronger candidates able to answer it correctly. These candidates were able to identify $x>0$ and $x<4$ as two of the three boundaries. Many included $y>0$. Many included the given equation, or $y=x+4$ often repeating these in a rearranged version or with different inequality signs.
(b) A minority of candidates found the correct area. There was a significant number who did not answer this part.

## Question 17

(a) The majority of candidates found the correct value for the angle. A few candidates made an arithmetic error subtracting $62^{\circ}$ from $90^{\circ}$ to get $18^{\circ}$ or $38^{\circ}$.
(b) Most candidates continued to find the required angle for this part. Some candidates gave the answer for $y$ as $62^{\circ}$ and others assumed angle $C B T$ was $90^{\circ}$ and subtracted $62^{\circ}$ and $90^{\circ}$ from $180^{\circ}$ or just calculated $180^{\circ}-62^{\circ}=118^{\circ}$.

## Question 18

(a) A very high proportion of candidates answered this correctly.
(b) Many candidates gave the correct expression. Incorrect answers were varied but the expression $n$ + 1 occurred regularly.
(c) Only stronger candidates wrote the correct expression. Incorrect answers were very varied.
(d) A minority of candidates gave the correct answer. Of these, some realised they needed to multiply the previous two expressions. Others started from the original diagram, generated a series from the totals of each row and then proceeded to show clear working using the formula
$a+(n-1) d_{1}+\frac{1}{2}(n-1)(n-2) d_{2}$. There was a significant number who did not answer this part.

## Question 19

The majority of candidates made a good attempt at drawing a histogram and accurate diagrams were often seen. Many candidates were awarded partial marks for at least three bars drawn in the correct place and/or at least three correct frequency densities seen. A common error was to draw the first bar the correct height but between 0 and 20 on the horizontal axis or to draw the last bar at a height of 1.2. A few candidates incorrectly left small gaps between the bars. Some divided all of the frequencies by 10.

## Question 20

(a) The majority of candidates did not give a fully correct description for the enlargement. Many were able to state the scale factor was 3 but either gave an incorrect centre of enlargement or did not mention the centre. Fewer candidates were awarded one mark for just the correct centre.
(b) A minority of candidates were able to draw the correct reflection of the triangle in the line $y=-x$. Many reflected the triangle in the x-axis or y-axis or rotated it $180^{\circ}$ about the origin.

## Question 21

Those candidates who knew how to find the gradient of the perpendicular line usually found the correct equation showing clear working. However, the majority did not know the method and did not score any marks. Many substituted either $x=8$, or $y=9$, or both of these values into the given equation or substituted
$(8,9)$ into $y=-\frac{1}{2} x+c$.

## Question 22

(a) Most candidates made a very good attempt at multiplying the matrices with many scoring at least partial marks. However, there were some arithmetic errors with the accuracy, including sign errors when finding the element -17 . Some multiplied the wrong combination of elements and a few applied the multiplication of $1 \times 0,2 \times 2,-4 \times 2$ and $3 \times-3$ to find each element.
(b) Few candidates gave the correct answer. Some realised the question involved the identity matrix but incorrectly gave this or an incorrect version of the identity matrix as the final answer. Some tried to find the inverse matrix for $B$.

## Question 23

(a) Most candidates were able to find the correct gradient which represented the deceleration of the train. Some gave the inaccurate decimal equivalent of $\frac{20}{60}$ as 0.3 . Some incorrect answers included 3 from $\frac{60}{20}$ or were the result of using the values 40 and 20 given on the graph.
(b) Most candidates found this part challenging and only a minority found the correct value for the time. Those who used a gradient method were usually successful. A few used similar triangles and these were often also correct. Some candidates who used the equation $v=u+$ at reached $t=30$ but did
not always realise they needed to add this on to 40 . However, many did not apply a valid method with some attempting to use the area under the graph or simply $\frac{40}{20}$ or $\frac{40}{20} \times 10$.
(c) Many candidates understood they needed to find the area under the graph and found the correct average speed. Those who did not realise the method relied on the area could make no progress with this question, with some dividing the speed of 20 by 100 .

## Question 24

(a) Most candidates were able to express 99 as the product of its prime factors. A few gave the answer as $3 \times 33$ or $9 \times 11$.
(b) (i) Nearly all candidates found this question very challenging and only a few successful answers were seen. After comparing $p$ and $q$ with the given expression for the lowest common multiple, candidates often gave the incorrect answers for $R$ as 5 or $5 \times 5$ or, more often, an expression that still contained powers of $n$. A significant number of candidates did not attempt this question.
(ii) In this part too, it was rare to award any marks. Many candidates began by copying the correct sum for $p$ and $q$ but were unable to proceed correctly, not realising they needed to factorise. A significant number did not attempt this question.

## Question 25

(a) Only stronger candidates were able to show the triangles were similar. Few realised the answer should only refer to pairs of equal angles with reasons. Some listed one pair of equal angles in addition to that given, but the reason for their equality was usually missing. Most incorrect answers referred to the lengths of sides or one triangle being an enlargement of the other.
(b) A minority of candidates were able to use the ratio of sides to calculate the missing length. A common error was to use the ratio of the incorrect pairs $\frac{A P}{A B}=\frac{A Q}{A C}$ resulting in $A P=16$. A few tried to solve the problem using Pythagoras' Theorem.
(c) Very few correct responses were seen. A few thought the area of triangle $A P Q$ was 3 times the area of $A B C$ so gave the answer as $3 x-x=2 x$. In many cases formulae for the area of a triangle or a trapezium were seen. A significant number of candidates did not attempt this question.

## MATHEMATICS D

## Paper 4024/12

Paper 1

## Key messages

To do well in this paper, candidates need to

- be familiar with the content of the entire syllabus,
- be competent at basic arithmetic,
- produce accurate graphs and diagrams,
- recall relevant formulas,
- set out their work in clear, logical steps.


## General comments

The majority of candidates were well prepared for most of the topics covered on this paper and most attempted all of the questions. Candidates across the ability range were able to demonstrate their knowledge with some questions accessible to all and several that offered challenge to the strongest candidates.

Most candidates demonstrated good skills in basic algebra and manipulation of fractions. Many would have benefited from a greater understanding of nets, graphical inequalities, transformations and matrices.
Candidates need more experience in answering questions where the mathematics required is not obvious, such as Questions 8 and 23.

In most cases, work was well presented with the method clearly shown and the answers transferred correctly to the answer line. Most candidates used correct geometrical equipment to produce neat, accurate diagrams. Candidates should show their working out in the answer space for each question, rather than on blank pages or additional sheets, so that partial marks can be awarded for a correct method seen if the answer is not correct. If an answer or the working is corrected, this should be crossed out and replaced rather than overwritten because overwriting often makes the candidate's intention unclear. Some candidates did not write figures clearly and in some cases it was hard to distinguish between 1,4 and 7 ; between 3 and 5 ; or between 0,6 and 8 .

Some candidates made mistakes in basic arithmetic. Errors were common in simple multiplications, for example calculating $3 \times 3$ as 6 . Other arithmetic errors involve incorrect simplification of fractions, incorrect place value when using decimals and slips in signs when using directed numbers. Candidates would have benefited from ensuring that their examination preparation includes reinforcement of basic arithmetic skills and from using any time available at the end of the examination in checking their work for these types of error.

## Comments on specific questions

## Question 1

(a) Most candidates were able to subtract the fractions correctly. Some reached the correct answer of $\frac{2}{15}$ and then incorrectly converted this to $7 \frac{1}{2}$ and others attempted to convert to a decimal which was not required. A minority of candidates subtracted the numbers on the numerator and the denominator leading to $\frac{2}{2}$ rather than converting to two fractions with a common denominator.
(b) This part was well answered. The most common error was to place the decimal point incorrectly leading to the answer 5.4 rather than 0.54 . Some candidates made arithmetic errors when multiplying 27 by 2 .

## Question 2

The method to find the fraction halfway between $\frac{3}{5}$ and $\frac{5}{7}$ is to use a common denominator and add the two fractions to give $\frac{46}{35}$, then divide this by 2 . Many candidates were able to find the correct equivalents with a common denominator and some added correctly, but the final stage of dividing by 2 was more difficult for some candidates. Some gave an answer of $\frac{46}{35}$ with no attempt to divide, others multiplied by 2 and some added incorrectly to give $\frac{46}{70}$. Some candidates subtracted the two fractions to give $\frac{4}{35}$ but did not know how to use this result to find the fraction required. Some candidates multiplied the two given fractions and others gave the answer $\frac{4}{6}$ or $\frac{2}{3}$, a result of forming a fraction using the numbers halfway between the two numerators and the two denominators. Some candidates converted the two fractions to decimals and attempted to find the value halfway between these. This approach is appropriate in some problems of this type, but as $\frac{5}{7}$ does not give a terminating decimal, it was not appropriate in this case.

## Question 3

(a) Many candidates were able to factorise this correctly. The most common errors were $4 t(3 t-t)$ and the incomplete factorisation of $2 t(6 t-2)$.
(b) This part was challenging for many candidates who could not identify that they had been given a partially factorised expression. It was common to see an incorrect answer of $(a+b)(x-y)$
because candidates had not seen that the second bracket in the given expression was $(y-x)$. Some candidates did not understand the term 'factorise' and expanded the brackets to give a fourterm answer.
(c) Many candidates factorised this expression correctly, although some answers such $(x-1)(x+3)$ or $(x-3)(x-2)$ were given. A small number of candidates attempted to solve the equation which was not acceptable.

## Question 4

Most candidates were able to order at least three of these values correctly. The length that produced the most errors was $4 \frac{1}{3} \mathrm{~m}$ which was often taken to be 430 cm or 433 cm . Some candidates just considered the units when ordering, leading to the answer $4340 \mathrm{~mm}, 433 \mathrm{~cm}, 4 \frac{1}{3} \mathrm{~m}, 0.043 \mathrm{~km}$. Others ordered the values without considering the units leading to $0.043 \mathrm{~km}, 433 \mathrm{~cm}, 4 \frac{1}{3} \mathrm{~m}, 4340 \mathrm{~mm}$. Most candidates used the original values in their answer rather than converted values.

## Question 5

Some candidates identified the correct method required for this question. Then, having reached the correct answer of 400 per cent, decided the answer must be incorrect and restarted with a calculation of $\$ 160$ as a percentage of $\$ 200$ which led to the common incorrect answer of 80 per cent. Of those candidates who divided by 40 rather than 200, many reached the correct answer. Other common incorrect answers were 4, where candidates did not multiply by 100 , or 500 per cent, where candidates did not subtract 100 per cent to give the percentage profit. Other errors were to find $\$ 40$ as a percentage of $\$ 200$ or to give the profit of $\$ 160$ as the answer rather than the percentage profit.

## Question 6

(a) Some candidates calculated the range correctly. A common error was to write the answer as a range of values such as -4.6 to -0.7 rather than to evaluate the difference as 3.9 . Some candidates made arithmetic errors in the subtraction leading to answers such as -5.3.
(b) Many candidates answered this part correctly, with some listing the three correct values which was accepted.

## Question 7

Many candidates were able to round the given numbers to one significant figure correctly and then correctly complete the calculation. Some gave the answer as an unsimplified fraction rather than 0.6 or $\frac{3}{5}$. Some place value errors were seen when attempting to simplify the calculation, usually when calculating $200 \times 0.3$ or $36 \div 60$. Some candidates were not able to round to one significant figure and 210 or 212 were seen in place of 200 or 0 in place of 0.3 . A minority of candidates attempted long multiplication and long division to work out the exact answer to the calculation which was not the intention of the question.

## Question 8

This question was challenging for many candidates. Candidates who realised that one section of the clock face was $360 \div 12=30$ were generally able to reach either $105^{\circ}$ or $75^{\circ}$ but they did not always continue to find the reflex angle required by the question. Some arithmetic errors in both the multiplication and addition were seen, for example $90+15=115$. Some candidates used $90 \div 4$ for each section on the clock face and others attempted to divide 360 by 2.30 pm using either 2.5 or 2.3 . The question asked for the answer to be calculated but measuring the angle with a protractor was common which led to answers which were close but incorrect because the diagram was not drawn to scale. A small number of candidates gave a time as the answer rather than an angle.

## Question 9

(a) Most candidates were able to expand the brackets and simplify correctly. The common errors were to evaluate $3 \times 3$ as 6 or to evaluate $-12+4$ as -16 .
(b) Many candidates did not know how to approach this question. Those that substituted $x=\frac{3 y}{4}$ often left their final answer as $\frac{7 y}{y}$ rather than simplifying this to the required numerical value of 7 . A common error was to replace $x$ with 4 and $y$ with 3 leading to the answer $\frac{35}{3}$.

## Question 10

Many candidates were able to solve these simultaneous equations correctly. Use of the substitution method led to fewer errors than the use of the elimination method on this occasion. Candidates who used the substitution method often rearranged the second equation to $y=5-4 x$, substituted into the first equation and reached the correct solutions. However, substitutions involving fractions, such as $y=\frac{3 x-12}{2}$, were more likely to lead to errors. Those candidates who used an elimination method usually multiplied the second equation by 2 to give $8 x+2 y=10$ and often reached the correct solutions, although some subtracted rather than added the two equations. Those who multiplied the first equation by 4 and the second by 3 sometimes made sign errors leading to $11 y=33$ rather than $-11 y=33$.

## Question 11

(a) Most candidates answered correctly. The most common errors were the answers $3.4 \times 10^{-5}$ or $34 \times 10^{5}$.
(b) Many candidates could simplify the expression to $\frac{1}{2 \times 10^{14}}$ but then this was often written as $0.5 \times 10^{14}$ or $2 \times 10^{14}$ or $2 \times 10^{-14}$. Candidates who correctly changed the expression to $0.5 \times 10^{-14}$ often correctly converted this to standard form although some moved the decimal point in the wrong direction to give an answer of $5 \times 10^{-13}$. Candidates should take care when writing their final answer on the answer line. Some made transcription errors, either omitting the negative sign in the power or writing -5 in place of -15 . Some candidates used the laws of indices incorrectly and added the powers rather than subtracting when carrying out the division.
(c) Many candidates found it difficult to answer this question because of the unknown power in the standard form values. A minority of candidates were able to manipulate the given equation to $7 \times 10^{a}-0.3 \times 10^{a}$ and then found $k$ using $7-0.3=6.7$. It was more common to assume that the given values could be subtracted ignoring the standard form element, and the answer 4 was common.

## Question 12

(a) Many candidates were able to cube the expression correctly. Common incorrect answers were $8 x^{5}$, where candidates had added the powers for the $x$ term, $2 x^{6}$, where candidates had just cubed $x^{2}$ and $6 x^{5}$, where candidates had multiplied 2 by 3 .
(b) Some candidates reached the correct answer, but many answers included either 9 or $t$ but not both. Many knew that they should invert the fraction to divide, but they treated $\frac{2}{3} t^{2}$ as $\frac{2}{3 t^{2}}$ leading to the answer $9 t^{5}$. Some arithmetic slips were seen in the simplification of $6 \times \frac{3}{2}$ and some answers were not fully simplified.

## Question 13

(a) Some candidates could interpret the set notation and gave the correct answer of 10. Common unacceptable answers were $\mathrm{n}(10), \mathrm{n}=10,\{10\}$ and lists of the elements of $P \cup Q$. Some candidates gave the answer 4, the intersection, rather than the union, of the sets.
(b) Many candidates were able to position the three letters correctly in the Venn diagram. The positioning of $p$ was found to be most straightforward. Some candidates placed $r$ in $A^{\prime} \cap B$ or inserted it twice to indicate $A^{\prime} \cap B$ and $A \cap B^{\prime}$. Some candidates inserted more than one letter in a region or wrote numbers in the regions, using the sets in (a) as $A$ and $B$.

## Question 14

The answers to this question often demonstrated that candidates are unfamiliar with the topic of nets.
(a) Candidates found it difficult to visualise the three dimensional solid from the net. The question referred to vertices, but many candidates only gave one answer, often either $G$ or $E$. Others gave more than two answers or gave an edge, such as JK.
(b) More candidates were able to answer this part correctly. Some gave the answer $A$ and $B$ or $A, B$ which each indicate two vertices rather than the edge $A B$. A common incorrect answer was the edge $H J$.
(c) The question identified the solid as a triangular prism, so the volume is found by multiplying the area of the triangular end by the length: $\frac{1}{2} \times 2 \times 2 \times 5$ for this prism. Candidates who did a volume calculation often either used $2 \times 2 \times 5$, the volume of a cuboid, or $\frac{1}{3} \times 2 \times 2 \times 5$, the volume of a pyramid. Many candidates attempted to find the surface area of the prism rather than its volume.

## Question 15

(a) Many candidates were able to calculate the probability correctly. Some arithmetic errors such as $\frac{3}{7} \times \frac{2}{5}=\frac{5}{35}$ or $\frac{6}{30}$ were seen. Some candidates added the probabilities instead of multiplying them.
(b) As the second bag did not contain any white beads, the probability that both are white is 0 . Some candidates were able to give this answer, but the answer $\frac{4}{7}$ was also common, the probability of taking a white bead from the first bag.
(c) Candidates who had answered (a) correctly were often also able to answer this part correctly. Common errors here were to calculate the probability that at least one ball is red rather than exactly one is red or to add their answer for both red to the probability of white, red.

## Question 16

Candidates who understood the meaning of frequency density usually answered both parts of this question correctly.
(a) The most common error was an answer of 20 which resulted from identifying that the frequencies given in the table were 5 times the frequency densities for the first two bars and not recognising that the width of the third bar was double the width of the first two.
(b) A common error in this part was again to use the multiplier 5 and draw a bar of height 6 rather than 2, but some candidates used a correct frequency density with a bar width of 20 rather than 15.

## Question 17

(a) Many candidates measured the angle accurately although some gave the answer $30^{\circ}$ rather than the correct bearing of $330^{\circ}$. Some misread the question and gave the bearing of $C$ from $A$ rather than $B$ from $A$. A minority of candidates did not understand the term bearing and gave a length, usually 9 cm , as the answer.
(b) (i) Many candidates understood that the required locus was the perpendicular bisector of $B C$ and used arcs to construct it accurately. Most bisectors crossed the triangle completely, as required, and it was acceptable to extend the bisector outside the triangle to assist with the accurate construction. Candidates needed to use two pairs of arcs which should not be deleted. Some bisectors showed only one pair of arcs or no arcs at all. Some lines were incorrectly drawn from a point on $B C$ to vertex $A$ and others constructed a bisector of angle $A$ or $C$.
(ii) Candidates who had attempted the correct bisector usually shaded the correct region, although some used one of their arcs as the boundary of the region rather than continuing the shading to the intersection of the bisector with $A C$. Others shaded areas for which the ship would be nearer to $B C$ than to $A C$ or nearer to $B$ than $C$.

## Question 18

(a) Most candidates gave the correct answer. The most common incorrect answer was 35 minutes, the time from leaving home to leaving the shop.
(b) (i) Most candidates drew the correct line, although some stopped where the two lines intersected rather than continuing to where he reached the shop.
(ii) Many correct answers were given in this part. A common incorrect answer was 800 m , the distance from home rather than from the shop.
(c) Candidates found this part of the question more challenging. Many identified that she had travelled 1200 m in 15 minutes and indicated that they needed to calculate $1200 \div 15$. Some correctly evaluated this as 80 metres per minute but were then unable to correctly convert this to $\mathrm{km} / \mathrm{h}$, demonstrating confusion about whether to multiply or divide by 1000 and 60 . Candidates who
converted to 1.2 km and $\frac{1}{4}$ hour before dividing were more successful with $1.2 \div \frac{1}{4}$ being transformed to $1.2 \times 4$ leading to the correct answer 4.8. Some candidates were unable to simplify $\frac{15}{60}$ correctly and others were unable to divide by a fraction. A minority of candidates attempted to multiply distance by time, and some used an incorrect length of time such as 35 minutes or a time of day such as 8.30 in their calculation.

## Question 19

(a) (i) The majority of candidates identified that the transformation was a reflection and some also gave the equation of the mirror line correctly as either the $x$-axis or $y=0$. Common errors were to give the mirror line as the $y$-axis or $x=0$ or occasionally $y=x$. Some candidates made no attempt to give the equation of the line, described the transformation as a mirror image or gave additional incorrect conditions to describe the transformation such as centre $(0,0)$.
(ii) This part of the question was challenging with few correct answers seen. Some stronger candidates gave a partially correct matrix or the matrix for a reflection in the $y$-axis. Common errors were to give a 2 by 3 matrix containing vertices of one of the triangles, the unit matrix, or a 2 by 1 matrix.
(b) The strongest answers to this part were from candidates who had drawn rays from the vertices of $A$ through the point $P$ and used these to determine the position of the triangle using their understanding that the negative scale factor meant that the triangle would be on the opposite side of point $P$ from triangle $A$. Some candidates drew correct construction lines but used point $P$ as one of the vertices of the enlargement. Many candidates used a scale factor of $+\frac{1}{2}, 2$ or -2 .

## Question 20

(a) Most candidates were able to continue the pattern and draw the correct rectangle, although some drew a $6 \times 5$ or $8 \times 5$ rectangle and some did not draw a diagram.
(b) (i) Again, most candidates answered this part correctly. The most common errors were the result of incorrect multiplications in the bottom row.
(ii) Many candidates could write down the correct expression for this sequence with a small minority giving a numerical value.
(iii) Candidates had more difficulty identifying the correct expression in this part. A common error was to use the difference of 2 to give the expression $n+2$ rather than $2 n-1$.
(iv) This part was challenging for many candidates. Some candidates understood that the answer in this part was the product of the previous two expressions and so gave a correct answer. Some omitted brackets in their product and others left the answer as pq rather than writing this in terms of $n$ as required. Some candidates did not use the previous answers and tried, not always successfully, to find a general term for the sequence $2,9,20,35$ using a formula they had learned or a difference method.

## Question 21

(a) Some candidates identified that the equations of the two boundary lines were $y=2$ and $y=2 x$ and used these to give at least one correct inequality, usually $y>2$. Some did not identify that the $y$-intercept the equation of line $A C$ was 0 and used equations of the form $y=2 x+c$. Some candidates included inequalities such as $y<8, x>1$ or $x<13$ and others used $2 x+3 y=32$ with alternative inequality signs.
(b) Only a small number of candidates were able to state the two correct values of $k$ in this part, with many unable to understand the term integer. It was quite common to see substitution of $y=7$ into $2 x+3 y=32$ leading to the answer 5.5 and less common to see substitution into $y=2 x$ leading to a second answer of 3.5. Some candidates gave a list of values, often 3, 4 and 5 with no method shown leading to them.

## Question 22

(a) Many candidates identified the isosceles triangle $O A B$ and reached the correct answer. Some identified the incorrect pair of equal angles, some calculated $180-48=132$ only and others made arithmetic slips such as $180-48=32$. Some candidates used angles in the same segment incorrectly to give either angle $O B A$ or angle $O A B=54^{\circ}$.
(b) Many candidates were able to use 'angle at the centre is twice the angle at the circumference' to give the correct answer here. Common incorrect answers included 180-108=72 and $54 \div 2=27$.
(c) Some candidates were able to identify that this part required use of the angle fact 'opposite angles in a cyclic quadrilateral sum to $180^{\circ}$. Common incorrect answers were 54 and $90-54=36$.

## Question 23

Many candidates were unable to identify a suitable strategy to use to solve this problem. Those who understood that there was a right angle at $T$ and identified that $O Q$ was the sum of the two radii $(4+x)$ generally used Pythagoras's theorem to set up a correct equation and often went on to reach the correct answer, although some were unable to simplify $\frac{84}{8}$ correctly. The most common error in using this approach was to expand $(4+x)^{2}$ as $16+x^{2}$ rather than $16+8 x+x^{2}$. Some candidates identified $O P$ as 4 and $P Q$ as $x$, usually on the diagram, but could make no further progress. It was common to see the answer 10 from the assumption that $A O=Q T$, or to see some attempt to use similar triangles or areas of circles.

## Question 24

(a) Some candidates understood that they needed to find the product of two matrices and reached the correct answer or a matrix with at least two of the elements correct. Many candidates were misled by the required matrix being $A^{2}$ and squared the individual elements leading to $\left(\begin{array}{ll}4 & 1 \\ 9 & 4\end{array}\right)$, or a variant of this including errors such as $(-3)^{2}=-9$.
(b) This part was challenging for many candidates. Some candidates who had answered (a) correctly understood that the answer here could be found using (0 $\left.\begin{array}{ll}0 & 2\end{array}\right)\left(\begin{array}{cc}2 & 1 \\ -3 & -2\end{array}\right)$. Many did not see this and attempted to find the inverse of $\left(\begin{array}{cc}2 & 1 \\ -3 & -2\end{array}\right)$ in which case it was common to see an incorrect determinant of 7 or -7 resulting from errors in manipulation of the negative numbers. Some candidates set up a pair of simultaneous equations $2 a-3 b=0$ and $a-2 b=2$ then solved these to reach the correct answer. It was common to see an attempt to evaluate $\left(\begin{array}{ll}0 & 2\end{array}\right) \div\left(\begin{array}{cc}2 & 1 \\ -3 & -2\end{array}\right)$ resulting from an incorrect rearrangement of the given equation.

## MATHEMATICS D

## Paper 4024/021

Paper 2

## Key messages

Candidates should use a suitable degree of accuracy in their working. Final answers should be rounded correct to three significant figures where appropriate or to the degree of accuracy specified in the question.

It is important for candidates to carefully read the information given in a question. For example, in Question 2(a), the number of potatoes was given in the stem of the question. A further example may be found in the stem to Question 5 which states the shape is a prism with a trapezium as its cross-section.

Candidates should set their work out logically and clearly, showing all stages of working leading to the answer or required result, in particular where a number of steps are required (Question 5(a), 6(c)(i) and 7(b)).

## General comments

In some places, candidates gave an inaccurate final answer due to inappropriate rounding of intermediate results. Typically, intermediate values should be rounded to at least one more significant figure than given in the question. It is important that candidates retain sufficient figures in their working and only round their final answer to three significant figures.

## Comments on specific questions

## Question 1

(a) This question was usually answered well, with most candidates answering correctly, although as the answer was exact in monetary terms, it should be noted the answer should not have been rounded to 3 significant figures, and instead left as an exact answer in dollars and cents. Just a small proportion of responses forgot to deduct the discount from the overall cost, or did not realise to multiply by 2 for the holiday for two people.
(b) This reverse percentage question was answered well by many candidates. Those responses which did not use a reverse percentage method tended to either reduce or increase $\$ 241.50$ by 15 per cent, neither of which scored any marks.
(c) This was a very well answered question, with many responses gaining full marks. The main error was in dividing $8(00)$ by 29.6 rather than the other way round.
(d) This was also answered well with most candidates gaining full marks. There was some confusion about whether to multiply or divide by the exchange rates and this tended to be where marks were not gained. To aid candidates in answering questions such as this, a ratio table can be used with dollar and euro headings which can help to improve understanding.

## Question 2

(a) (i)(a) The stem of the question, before the graph, stated that 60 potatoes are represented in the diagram, so the answer could be found by reading the mass at a cumulative frequency of 30 (from $60 \div 2$ ). A number of responses used a total of 70 potatoes, suggesting that information in the question had been overlooked, and the graph had not been examined closely enough. Some candidates used
only the mass axis to find the median, by using the extremes of 120 and 260 and finding a value halfway between, with no reference to the cumulative frequency.
(b) Similar issues occurred with the interquartile range as with the median in (a), that is, using a total of 70 potatoes rather than 60, or using only the mass axis and ignoring the cumulative frequency entirely.
(ii) Those candidates who were successful in (i) tended to continue to do well here. A common error was to find the masses above 220 and divide this figure by 260 , thus ignoring the frequency. The question required candidates to read the cumulative frequency at 220 (a value of 52), and therefore find there were 8 potatoes out of 60 that were sold as baking potatoes, from which they could find the required percentage.
(iii) As the question stated the potatoes that are more consistent in mass were required, candidates needed to think about how this applied to the statistics they had just calculated. "More consistent" implied the need for a smaller range of some type. The only statistical data known about both variety $A$ and $B$ were median and interquartile range, therefore it was the variety with the smaller interquartile range which Kali should have chosen. Candidates needed to state variety B should be chosen because it had a smaller interquartile range.
(b) This question was often answered correctly. Most candidates realised they needed to find the midpoint of each group and multiply by its frequency to begin to find the total of all 120 masses. However, there were responses where the midpoints were added, with no reference to frequency. There were also responses which ignored the mass entirely, and instead added the frequencies and divided by 5 . These last two methods gained no marks.
(c) A minority of candidates recognised this as a bounds question. A common error was to add 0.6 to $10 \times 2.5$. The question required candidates to find the upper bound of $2.5(2.55)$ and the upper bound of $0.6(0.605)$ before applying the calculation.

## Question 3

(a) (i) Most candidates scored full marks on this question, with clear intersecting arcs shown.
(ii) It was expected candidates would measure the vertical height and horizontal base of their triangle, and use the formula area of a triangle $=\frac{1}{2} \times$ base $\times$ height. A large number of responses measured angles and instead used the formula area of a triangle $=\frac{1}{2} a b \sin C$ which was also perfectly acceptable, but seemed more prone to error as the angle measured often did not correspond to the two sides used. There were also some complicated attempts to use Pythagoras' theorem or trigonometry. Candidates were asked to take suitable measurements to answer the question rather than an answer based solely on calculations.
(b) (i) Most candidates recognised that the sine rule was required. Some candidates did not gain marks due to premature approximation with the sine of the angle. Other errors included candidates attempting to use the cosine rule, or using no trigonometry at all.
(ii) Here the cosine rule needed to be used, which it often was, although there were formula issues for some candidates. These consisted of not being able to remember the cosine rule formula for finding an angle, or being unable to correctly rearrange the cosine rule from the finding a side format. Premature approximation was again an issue for some candidates with the final mark not awarded to some candidates due to this.

## Question 4

(a) The answer to this part was almost always correct.
(b) There were many good attempts at plotting the smooth curve and most candidates gained full marks in this part. Most sketches involved a correct smooth curve rather than line segments drawn with a ruler.

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(c) Few candidates were able to draw a tangent at the correct point and to use this to estimate the gradient of the curve. Others knew how to draw the tangent but did not use the correct method for the gradient or made errors in their calculation. There were a number of responses where the tangent was drawn at $y=2$ rather than at $x=2$.
(d) There were a number of responses which gave the correct answer through a trial and improvement method, but it should be noted the question asked candidates to use the graph to estimate the solution. It was expected that candidates would divide both sides of the given equation by 5 to give $\frac{4}{5} \times 2^{x}=1$. The left side of this is the same as the graph function just drawn, and so the solution could be found by reading the value of the graph when it is equal to the right side, that is when $y=1$.

## Question 5

(a) The most common misconception here was in thinking the prism was a cuboid, rather than having a cross-section which was a trapezium. Candidates should take care to read the information given in the question, rather than relying on one diagram to show all information. The formula for the area of a trapezium was not widely recalled by those who did realise this was needed. There was also some confusion between volume and total surface area.
(b) The misconception of a cuboid continued in this part. The most common error here was to find the area of the roof as $2.10 \times 1.55$ (thinking the sloping length of the roof was 1.55 ). Instead, Pythagoras' theorem was needed to find the sloping length. This could then be multiplied by 2.10 to calculate the area.
(c) There were again some premature approximation issues in finding the angle of elevation here. A large proportion of candidates did not know how to successfully attempt this part. The question firstly required the use of Pythagoras' theorem to find the distance AF. Trigonometry could then be used to find the angle $A F D$ in the right-angled triangle $A D F$.

## Question 6

(a) Many candidates were able to correctly solve the inequality, but there were a minority who having found $x>1.5$ correctly in their working, then wrote the final answer as 1.5 . It should be noted that the full answer of $x>1.5$ should have been written on the answer line. There were also some candidates who swapped around the inequality sign in their working, for no apparent reason, and finished with $x<1.5$ which gained just one mark.
(b) Almost all candidates successfully formed the pair of simultaneous equations and went on to find the correct solutions. Both the elimination method and the substitution method were used with roughly equal success.
(c) (i) There were many candidates who managed to eliminate the fractions and solve the equation correctly. However, there were also a large proportion of candidates who did not know how to add two fractions and did not realise both the left and right sides of the equation needed multiplying by the denominators in order to remove these.
(ii) Solving the quadratic equation was usually done well, although candidates should take care when some coefficients are negative. Candidates recognised the need for the use of the formula rather than factorisation, with the instruction in the question that answers were required to 2 decimal places.

## Question 7

(a) Candidates were reasonably successful at this circle theorem question, often finding angles $B O D$ and $B C D$ correctly. They did not always then realise that $O B C D$ was a quadrilateral to finally find angle $C D O$. Some candidates successfully found angle CDO by drawing the line OC and using two isosceles triangles after finding either of the angles $B O D$ or $B C D$.
(b) (i) As noted in the key messages, candidates should set their work out logically and clearly in a question such as this where there are a number of calculations to be done. Most candidates who
realised they were finding total surface area rather than volume found the area of one or both rectangles, and a smaller number also calculated the area of one or both sectors. Most had difficulty with the area of the curved side, thinking perhaps this was a rectangle, rather than needing to find a fraction of the circumference and multiply this by the height of 5.
(ii) The most common error in this part was in dividing the answer from (i) by either 27 or 28 . As the ratio given in the question is a ratio of volumes (not areas), firstly a ratio of sides is needed (found from $\sqrt[3]{27}=3$ ) and then a ratio of areas can be found from $3^{2}=9$. This finally gives the value which the answer from (i) should be divided by.

## Question 8

(a) Candidates found this question challenging. As the balls were not replaced in the bag, the denominators should both have been 35 (as in the second set of branches already given in the question). As no red balls had yet been taken, there were still $n$ red balls remaining, and so $35-n$ green balls were left.
(b) This part tended to be answered more successfully than (a), perhaps as it used probabilities already given on the tree diagram. Some candidates did more than just 'write down' as instructed, and attempted to simplify their answer. Some candidates added probabilities rather than multiplying them.
(c) Reading the instruction more carefully here may have helped some candidates gain more marks. Some candidates clearly thought that as a quadratic equation was presented to them in the question, it meant it needed solving (even though the next part of the question asks them to do just this). In this part, candidates needed to equate their answer from (b) and rearrange this to the given answer. This is a question where all stages of working should be clearly shown as the answer is quoted in the question.
(d) Those candidates who recognised the equation factorised tended to be more successful than those who used the quadratic formula to solve it. Again, candidates should take care when some coefficients are negative if using the quadratic formula. In general, working was shown by most candidates.
(e) As this part relied on previous answers, there was a significant proportion of candidates who omitted this part. Candidates needed to take the smaller of their two answers from (d) for the value of $n$. They should then have substituted this into the two probabilities for choosing a green ball from the tree diagram and multiplied these together.

## Question 9

(a) (i) A large number of responses to this part found $\overrightarrow{\mathrm{OH}}+\overrightarrow{\mathrm{OJ}}$ rather than $\overrightarrow{\mathrm{OJ}}-\overrightarrow{\mathrm{OH}}$. Other than this, there were some arithmetical errors but most candidates attempted the question.
(ii) Most candidates realised the use of Pythagoras' theorem was needed here. Care should be taken when subtracting and squaring negative numbers.
(iii) For those candidates who found $\overrightarrow{O H}+\overrightarrow{O J}$ in (i) all they were required to do here was to halve their answer, which many did successfully. Others were successful in gaining one or both components of the midpoint.
(b)(i)(a) Both this part and the next are intended as a lead in to the more complicated vector questions in (ii) and (iii). Candidates were expected to recognise that $\overrightarrow{F G}$ and $\overrightarrow{G B}$ were both equal to $p$ so the overall answer was $2 p$.
(b) Although fewer candidates answered this part than (i) there were more correct answers. There are many routes from $F$ to $E$ but $\overrightarrow{F G}+\overrightarrow{G E}$ was possibly the most straightforward. This led to a final answer of $p+q$.
(ii) Although few candidates gained full marks here, many found a correct vector route for $\overrightarrow{O X}$ along the diagram lines which achieved one mark. The easiest route to take was $\overrightarrow{O F}+\overrightarrow{F X}$ where $X$ is $\frac{3}{4}$ of the way along $\overline{F X}$.
(iii) A very small minority of candidates gave the correct answer here. The most common correct method was to use the fact that $\overrightarrow{O X}$ and $\overrightarrow{F Y}$ are parallel. Then, as $Y$ is on $B D, \overrightarrow{F Y}$ must be equal to $2 p+k q$. This meant by finding the multiplier from $\overrightarrow{O X}$ to $\overrightarrow{F Y}$, the vector $\overrightarrow{F Y}$ could be found. Finally $\overrightarrow{X Y}$ could be found from $\overrightarrow{X O}+q+\overrightarrow{F Y}$.

## Question 10

(a) The answer to this part was almost always correct. Answers could take any evaluated form, for example $2.2,2 \frac{1}{5}$ and $\frac{11}{5}$ were all acceptable.
(b) Many candidates gained 2 or 3 marks here, successfully understanding the notation was asking for the inverse function of $f(x)$. Common errors were not rewriting the final answer in terms of $x$, but instead leaving the answer in terms of $y$. Occasionally there were errors when rearranging with negatives.
(c) By far the most common error here was in misunderstanding the function notation. This resulted in a number of candidates finding $g(p)$ correctly, but then equating this to $f(p)$ multiplied by $(p+1)$ rather than to $\mathrm{f}(p+1)$. Those candidates who set up the initial equation correctly usually went on the find the correct value for $p$.

## Paper 4024/22

Paper 2

## Key messages

Candidates need to ensure they read each question carefully. For example; Question 1(c), candidates did not always give their answer to 2 decimal places; Question 1(d)(iii) candidates did not always answer using the two equations; Question 4(b) candidates did not recognise that it was the measurements that were to the nearest centimetre; Question 5(a) candidates did not always find the number of stamps that Sanjay has.

Premature approximation was seen on several questions, and instead candidates need to either store values in their calculator or have sufficient accuracy for values in their calculation that are then used to obtain the final answer.

## General comments

Candidates taking this exam attempted the majority of the questions, with varying degrees of success. Working was usually shown, particularly on the questions which specifically ask for this. Candidates generally answered Questions 1(a), 3(b) and 6(a) well. Questions 2(b)(iii), 7(b)(i), 7(b)(iii) and 10(d) were often more challenging.

## Comments on specific questions

## Question 1

(a) Many candidates were able to correctly calculate the total amount Marta paid. When full marks were not awarded, evidence was often seen of correct understanding of either the deposit or the monthly instalments. One common error was to think the cash price of the car needed to be included in addition to the deposit and the monthly payments.
(b) Many candidates were able to answer this reverse percentage question. The most common error was to find 112 per cent of the sale price, giving the answer $\$ 320.32$.
(c) Many candidates found this question challenging with methods ranging from finding the difference of the two rates, leading to the answer $\$ 1=€ 0.16$ or the product of the two rates leading to an answer of $\$ 1=€ 0.62$, to inverting the calculation, leading to the answer $\$ 1=€ 1.23$. Many did the calculation in more than one step and rounded their results prematurely getting an answer of $\$ 1=$ $€ 0.81$. Some candidates did not give their answer correct to 2 decimal places.
(d) While many candidates knew the correct formulae for both compound and simple interest few candidates gained full marks for this question. The most common error with the compound interest was to round the answer to full dollars giving the total as $\$ 1648$ rather than $\$ 1648.02$. Some candidates either did not know how to enter the calculation into their calculator or they did the calculation in several steps, which although a valid method, introduced many rounding errors. Many could not distinguish between the interest calculated and the original $\$ 1500$ principal, causing them to either double count the principal in the compound interest part or omit the principal from the simple interest calculation.

## Question 2

(a) (i) A majority of candidates used the correct cumulative frequencies, but not all were plotted at the upper end of the intervals. Most of the incorrect points were plotted at the mid-points but a few were plotted at the lower end of the intervals. Other incorrect responses included bar graphs, some for cumulative frequencies and some for frequencies, along with frequency polygons.
(ii)(a) If an increasing curve was seen, then candidates were generally successful in finding the median.
(b) Fewer candidates successfully found the interquartile range. Some candidates correctly found either the upper quartile or the lower quartile. Other candidates realised that the quartiles had cumulative frequencies of 20 and 60 , subtracted these to get 40 , sometimes giving this as the answer, and others read off at 40, not realising that it simply repeated the median.
(b) (i) Many candidates demonstrated a good understanding of the mean and obtained the correct value. However, there were some who gave their answer either to one decimal place or the nearest whole number. Some knew the correct method but an arithmetic error, usually $0 \times 24=24$ or $3 \times 32=64$, resulted in incorrect final answers. Typical errors included dividing the total frequency by 6 and dividing the correct total by 6 .
(ii) Many responses gave the probability of 3 or more errors. Others gave the probability of exactly 3 errors. Some candidates attempted to add the frequencies but did not get 160 despite this being given in the question.
(iii) This part proved even more challenging and only a few candidates answered correctly. Many found two probabilities, often incorrect, but were then unsure whether to add or multiply. It was common to see the two probabilities with denominators of 30 and 29 . Some assumed replacement and so $\frac{9}{256}$ was a common incorrect answer.

## Question 3

(a) Most candidates were able to complete the table correctly. However, some candidates gave the truncated value of 2.03 .
(b) Plots were generally accurate with reasonable attempts to join them with a smooth curve. The most common plotting errors were at $x=7$ followed by at $x=1$ and $x=1.5$. Some candidates were confused by the regularity of the $x$ points, causing them to miss a point and some struggled to plot values given to 2 decimal places using the scale of the grid. There were a few examples of straightline sections in the main curved part of the graph and a small number of bar charts or plots with no curves. Some drew rings around their plotted points or plotted large blobs.
(c) There were a high number of very good tangents drawn and many good attempts. Some tangents touched the curve on one side of $x=1$, but did not reach the point itself. Many candidates found the correct gradients by using the coordinates of two points rather than drawing a triangle and evaluating lengths. A common error was to give a positive gradient or to read the $y$-value at $x=1$ without drawing a tangent.
(d) (i) There were many good lines, often not extending from $y$-axis to $x$-axis, but long enough to cut their curve twice. Some candidates struggled to find points to plot with lots of writing in the working space but no correct points found. The simplest solutions seen were those with evaluated coordinates where $x=0$ and $y=0$. Not all lines were straight or ruled and plotted points were sometimes connected even if one seemed clearly out of line with the rest.
(ii) Most candidates who had drawn an intersecting line were able to give the correct $x$-value for their intersections. The left-hand point was more often incorrect because of the proximity of the tangent, with many candidates giving the intersection value with that rather than the curve.
(iii) Some candidates presented clear answers that were fully correct. Many who correctly combined the equations made one or more slips in rearranging but were still able to reach the point where they achieved $3 x^{2}$ with one other term correct. There was significant difficulty in dealing with the $2 / x$ term and sign errors in moving all non-zero terms to one side of the equation. A significant number of candidates did not attempt this question. Despite the instruction given, there were a number of candidates who chose to substitute their $x$-values from (ii).

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## Question 4

(a) (i) Most candidates had some idea of the steps required to answer the question. Some were successful but in a lot of cases errors resulted in an incorrect answer. Common errors included using incorrect formulae or incorrect values for the radius of the cylinder, quite often the value 3. When appropriate formulae were used, candidates were generally successful in rearranging to the correct form. Many did not realise that they needed to give the radius to a minimum of three decimal places. A small number used 7.86 in their responses to try and prove that the value was 7.86 .
(ii) There was a lot of confusion about the different parts of the surface area and how to find them. Most candidates were able to calculate the curved surface area of both the hemisphere and/or the curved surface area of the cylinder. Common errors usually involved incorrect formulae or use of incorrect values, usually for the radius of the cylinder, and an incorrect number of surface areas. Few candidates realised that the upper surface of the cylinder and the exposed flat surface of the hemisphere together were the same as the complete plane surface of the hemisphere.
(b) There were two distinct methods used in attempting this question. Many candidates realised that it was necessary to find the lower bound of each measurement first and then find the volume. However, many candidates then rounded these correct volumes to fewer figures. The second method was incorrect, calculating the volume first and then attempting to determine a lower bound for this volume.

## Question 5

(a) This part of the question proved challenging and a full range of responses were seen. A small majority were able to give a correct equation and solve it. After reaching a correct equation some candidates subtracted 7 from 108. Weaker candidates struggled to set up a correct equation, making errors with some of the individual terms such as $7-2 n, 2 n+7, n^{2}$, etc. and often missing out one or two terms in their equation. Some were able to solve their equation correctly but, in most cases, their answer was a non-integer and this was sometimes used to find the number of stamps for Sanjay. Several candidates went no further than solving the equation, giving the answer as 23.
(b) A majority of candidates displayed good algebraic techniques and obtained the correct answer. Common errors included incomplete cancelling, $\frac{6 v^{5}}{15}$, losing at least one of the factors, $\frac{v^{5}}{5}$, and multiplying $v^{3}$ and $v^{2}$ and getting $v^{6}$. Some candidates had difficulty in converting from a division to a multiplication and inverted the first fraction.
(c) Many candidates obtained the correct final answer. In a few cases, other candidates had both sets of factors correct but did not cancel the fraction to its simplest form. When attempts to factorise the numerator and the denominator were seen it was more common to see the numerator correct. With the denominator, common errors included the signs interchanged, $(3 x+2)(x-4)$, or the constants interchanged, $(3 x+4)(x-2)$. Many weaker candidates struggled and incorrect attempts were seen to cancel parts of the numerator and denominator independently.

## Question 6

(a) A large number of candidates got the correct answer for this question.
(b) Many candidates started correctly and reached the point $8-4 x>3$ without too much difficulty. However, some were then unable to deal correctly with the negative quantity of the $x$ term and had an answer with the wrong inequality sign, so $x>1.25$ was a common wrong answer. A final answer of simply 1.25 was also common, with candidates using an equality sign throughout or dropping their inequality sign when putting their otherwise correct answer in the answer space.
(c) Many candidates were able to find the inverse function correctly. For those candidates who understood the basic principle of the method, the most common error was not to deal with the negative value of $x$ correctly. A few did not understand the inverse notation and started working with reciprocals.
(d) Many candidates did not understand this question and simply tried to rearrange the given equation to find $p$. Some had equations containing both $x$ and $p$, such as $8-x(p)=7-\frac{3 x}{5}(2 p+1)$ or similar. For those who knew what was required, the most common errors were not applying the minus sign to the second term of the fraction, leading to an answer of $\frac{1}{7}$, or forgetting to multiply the 7 by 5 .

## Question 7

(a) (i) A minority of candidates were able to find the correct value of $n$. Some candidates were able to work out the interior or exterior angle of the polygon, but not all were able to use this correctly. Attempts were seen to use $\frac{180(n-2)}{n}$. However, the expression was frequently equated to 24 and not 156. Many answers were decimals as candidates failed to realise that the value of $n$ required an integer.
(ii) Few candidates achieved a correct answer in this part. A common incorrect answer was 24. Often the magnitude of the answer given did not correspond to the size of the angle in the answer space. Many candidates did not attempt this part of the question.
(b) (i) Few candidates were able to give a sufficient argument regarding the similarity of the triangles. However, many were able to show some understanding of what was required by stating angles that were equal and some with correct reasons. Occasionally reference was made to angles which were not in either of the given triangles.
(ii) More correct answers were seen in this part of the question. Common errors included 2:1, 1:2 and 3:2.
(iii) Very few correct answers were seen here. A few candidates found the ratio of the two triangles from (i), giving their answer as either 1:9 or 9:1. Some candidates knew that the area ratio was the square of the linear ratio and so squared their ratio from (ii). Occasionally an answer of $\frac{1}{2}: 6$ or equivalent was seen when candidates worked with $\frac{1}{2} \times 1 \times 1: 3 \times 2$. A number of candidates did not attempt this question.

## Question 8

(a) (i) A correct method was often seen but not all candidates were able to evaluate this correctly as many struggled with $(-6)^{2}$ resulting in a magnitude of 8 being a common wrong answer. Some candidates used an incorrect formula and subtracted the values, while others tried to use the coordinates of $H$ to obtain their answer.
(ii) Slightly more candidates were able to find the correct coordinates of $K$. However, there were some who, having found the vector $H K$, then made an arithmetic mistake or subtracted the coordinates of $H$.
(b) (i) Few candidates answered this question correctly. The most common error was adding $5 \mathbf{q}$ rather than $\mathbf{q}$, indicating use of the ratios rather than the diagram.
(ii) A similar number of candidates were able to give the correct vector here while others were often able to state a correct vector route.
(iii) This question was challenging for many candidates. The fraction of vector $\mathbf{q}$ or the direction of the vector $\mathbf{q}$ was often incorrect. The diagram was well used by candidates to find a correct route from $E$ to $X$.

## Question 9

(a) (i) Candidates demonstrated a good understanding of the requirement to use the sine rule to answer this question. However, some candidates truncated their angle either to $41^{\circ}$ or $41.3^{\circ}$. In addition, the accuracy of the angle was lost by some candidates who chose to truncate the sine of the angle to 0.66 or 0.661 .
(ii) Many candidates recognised the need to use the formula $\frac{1}{2} a b \sin C$ to find the required area. However, not all used the correct angle and use of 79 or 41.4 was common. Some candidates recognised the need to calculate the third angle of the triangle first but some then rounded the angle to the nearest degree. A common error in approach was to assume angle $C$ is a right angle to calculate the area of the triangle, resulting in 30.4 as the answer.
(b) (i) The angle was often correctly found using the explicit form of the cosine rule. However, some candidates who chose to use this approach did not always state this form correctly resulting in wrong angles seen. Those who chose to start with the implicit form of the cosine rule and substitute the values sometimes struggled to rearrange it accurately resulting in the wrong value for the cosine of the angle. Some candidates used an incorrect approach by using the sine rule and making wrong assumptions.
(ii) Some candidates understood what was required to calculate the angle of elevation. Common mistakes were to give the angle as 51.3 or to add $90^{\circ}$ to the angle of elevation.

## Question 10

(a) The majority of candidates were able to show the expression for the average speed.
(b) Candidates who made an attempt at connecting $\frac{600}{x-25}, \frac{1800}{x}$ and 3 into an equation showed some good algebraic manipulation to attempt to get to the correct equation. A number of candidates did not answer this question.
(c) The quadratic formula was well used by many candidates but not all of them managed to give answers correct to 1 decimal place with the most common mistake being answers given correct to 3 significant figures. Mistakes were also seen in the formula but the correct solutions were shown, indicating candidates were using their calculators to solve a quadratic equation.
(d) This question offered a significant challenge to many candidates. Few candidates successfully calculated the total time for the journey. The most common error was to work out the average speed for each part of the journey and then find their average. The definition of average speed is total distance divided by total time and this approach was not seen in many answers as the total distance of 40 was rarely shown.

